Hayfield Lane Calculation Policy

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Aims

The National Curriculum for Mathematics aims to ensure that all pupils:

- *become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.*

At Hayfield Lane we will therefore move our children beyond simple memorisation of facts and rules and ensure they have a deep-rooted understanding of the different branches of Mathematics, and how they connect together. For our children to become fluent, they need to understand the meaning of addition and its inverse relationship with subtraction; know fluently a variety of number facts such as number bonds to 1, 10 and 100, and the commutativity of these; and a deep understanding of our place value system, how the numbers are structured within it and how they behave in addition.

The National Curriculum for Mathematics aims to ensure that all pupils:

- ***reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.*

At Hayfield Lane we will therefore think carefully about what we want our children to think, notice and understand about the mathematics involved in the learning activities. We will help our children to get underneath what is going on, to make links, and to generalise their understanding. Mathematical talk will play a big part in our lessons and we will use a range of vocabulary.

The National Curriculum for Mathematics aims to ensure that all pupils:

- *can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*

At Hayfield Lane we will therefore incorporate a wide range of investigations and problem solving activities into our lessons to enable children to think mathematically. We will emphasise the importance of being stuck, having another go and trying different approaches.

At every stage of calculation, we need to switch between the **concrete**, **pictorial** and **abstract** (CPA) as appropriate.

Concrete – real life objects, practical resources
Pictorial – drawing pictures of practical resources, bar models
Abstract – number lines, equations with numbers and symbols, bar models

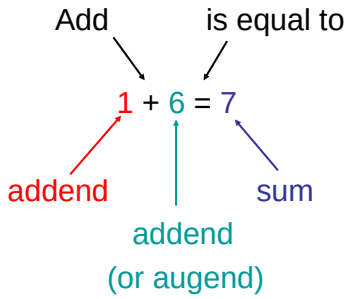
Vocabulary – Addition and Subtraction

"Mathematical language is crucial to children's development of thinking. If children don't have the vocabulary to talk about division, or perimeters, or numerical difference, they cannot make progress in understanding these areas of mathematical knowledge."
Mathematical Vocabulary, DfE 2000

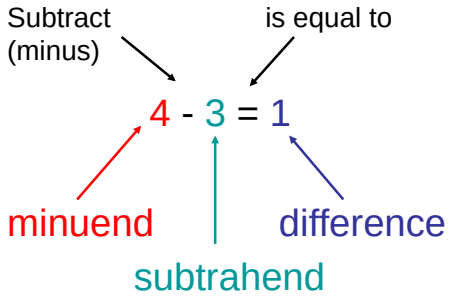
The National Curriculum for Mathematics is very clear that the correct use of mathematical language is central to a meaningful and deep understanding. Having a wide vocabulary of mathematical terminology available is essential for mathematical thinking and reasoning – we think in the same words that we speak. It is not enough for children to simply hear mathematical words; they need to 'feel' them in their own mouths. Therefore when introducing new vocabulary, everyone needs to repeat it out loud (in KS1 we use the principals of 'my turn, your turn from Read Write Inc). It is also essential that new vocabulary is explained carefully and introduced alongside relevant real life contexts, practical resources or pictures so that children really understand.

Teachers need to have high expectations and only accept what is correct.

✓	×
ones	units
is equal to	equals
zero	oh (the letter O)

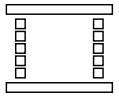


Key Vocabulary
Equality, inequality, inverse
Addition, add, plus, make, altogether
Total, sum, increase, combine
Subtraction, subtract, minus
Difference, take away, leave, decrease
More than, less than, X more, x less
Digit, tens, ones,
(near) multiple of 10
Partition, regroup, exchange
Also see vocabulary guidance document



Note: 'take away' is just one strategy for subtracting

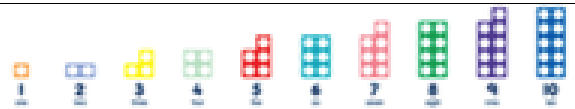
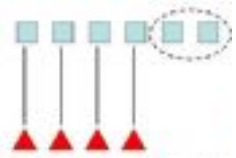
An **equation** is a mathematical statement, in symbols, that says two things are equivalent or the same [number sentence] e.g. 2=10-8, 9-3=6, 5=5

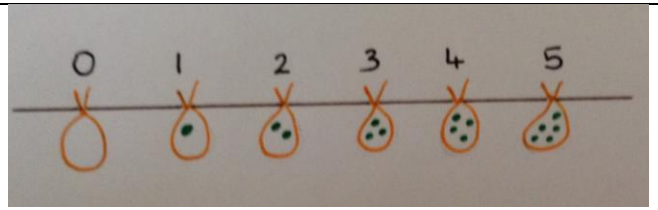


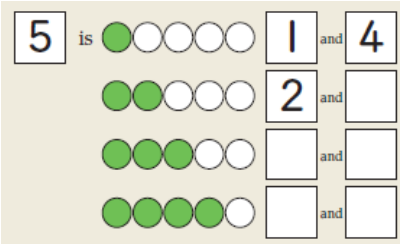
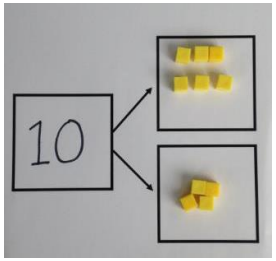
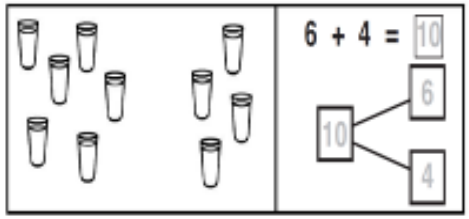
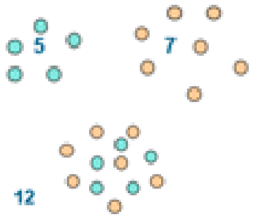
An **inequality** is a mathematical statement that two things are not the same e.g. 3 < 10 - 9, 12 – 8 > 2 5>2

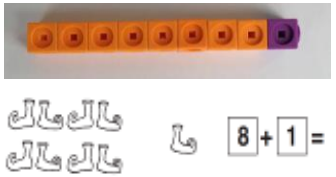
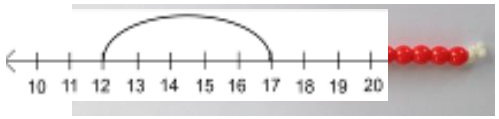
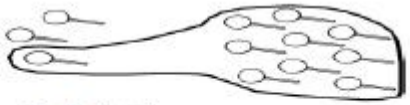
2<5

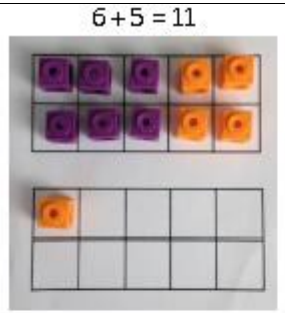
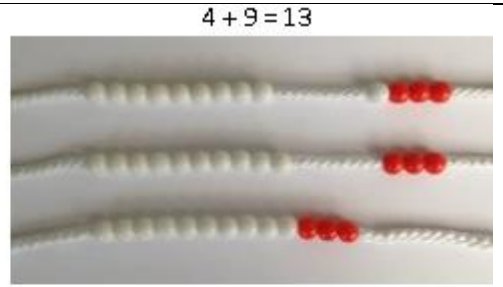
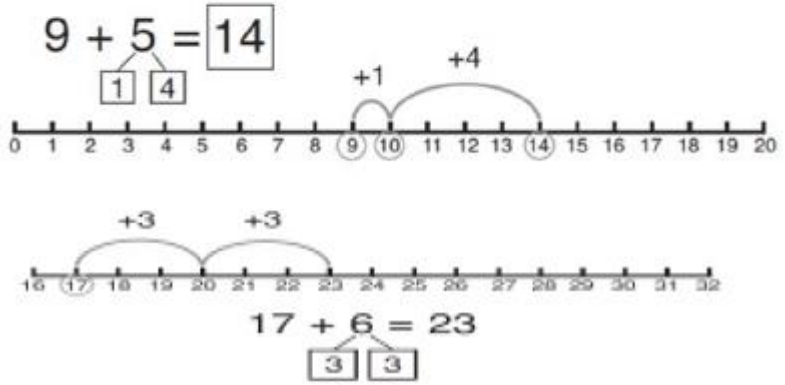
Progression in the Early Stages (mainly EYFS and KS1) ADDITION AND SUBTRACTION

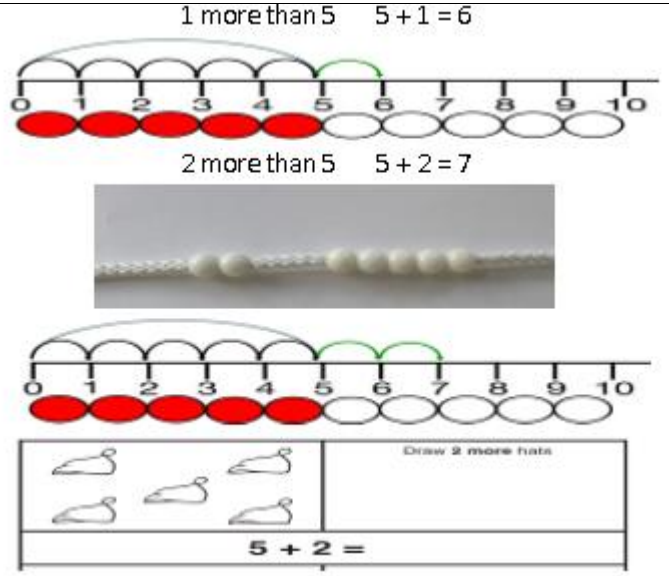
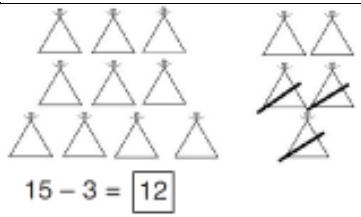
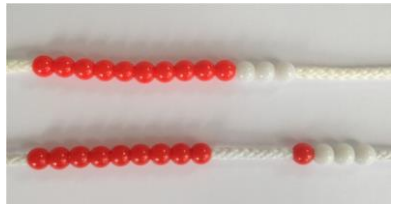
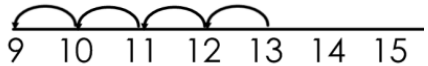
Strategy	Notes	Representations
Making groups and numbers	Recite and count the number words e.g. "two" Read and write the numbers e.g. 2	 Make a one-to-one correspondence between two groups of elements e.g.  Meaning of zero 0 as an empty set e.g.

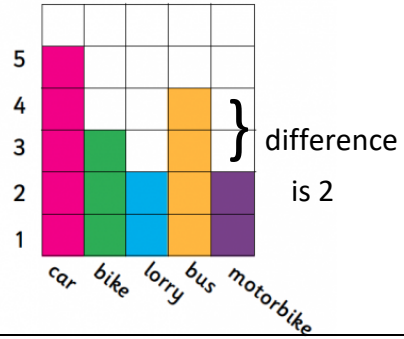
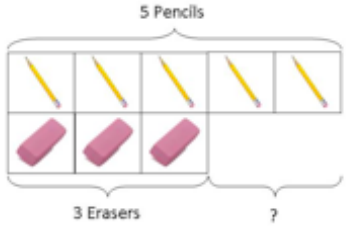
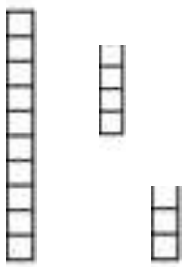
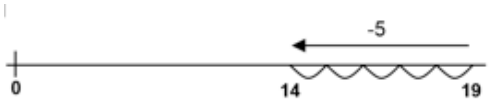
		
Ordinal numbers	Match cardinal and ordinal numbers	Represent order and position using numbers 

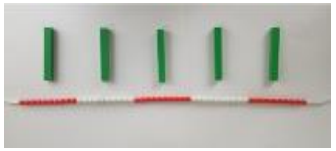
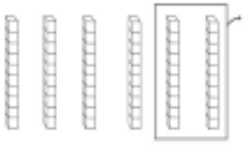
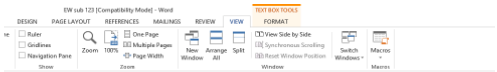
Composing and decomposing numbers (number bonds)	Teach addition and subtraction consecutively so that pupils make the links. Pupils will use the part-part-whole model to identify the inverse link between addition and subtraction. Pupils will develop the understanding of commutativity of addition as they become aware that the parts will make the whole in any order.	Partition numbers in different ways. Consider a number as a sum or difference of two other numbers e.g.    6 + 4 =10 4 + 6 = 10 10 – 6 = 4 10 – 4 = 6
Adding together: the meaning of addition as combination	Encourage children to read equations aloud in different ways. Include addition that involves 0.	   Numicon shapes are great for adding without counting – we want children to be able to 'see' numbers.

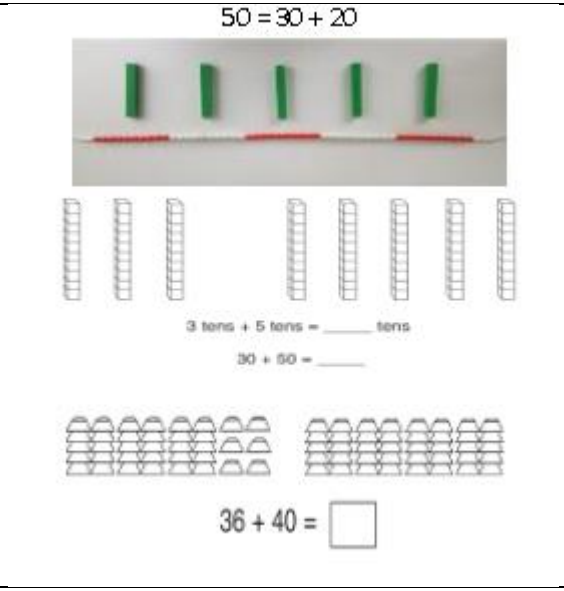
Adding more: the meaning of addition as increase	Include addition that involves 0.	Number lines can be used alongside practical apparatus to show 'counting on': 8 + 1 = 9  17 = 12 + 5 
Regrouping ten ones to make ten	An essential skill that will support column addition later	 3+9=12 

'Making ten' strategy	Pupils should be encouraged to begin with the greater number and use the smaller number to make ten. The colours of the beads on the bead string make it clear how many more need to be added to make ten. The empty spaces on the ten frame make it clear how many more are needed to make ten.	  

Adding 1, 2, 3 more	Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as: '1 more than 5 is equal to 6' '2 more than 5 is 7' '8 is 3 more than 5'	
What is left? The meaning of subtraction as decrease	When 'taking away' is first introduced, the concrete representation should be based upon the picture – place real objects on top of pictures. Include subtraction that involves 0.	  13 – 4 = 9  6 – 2 = 4 

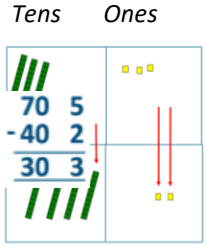
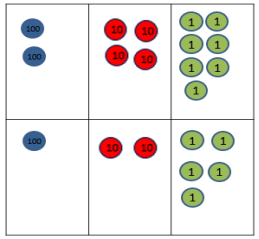
What is the difference? The meaning of subtraction as difference	Record equations with the equals sign in different place. Include subtractions that involve 0.	 difference is 2  
Add a 1 digit number to a 2 digit number; Subtract a 1 digit number from a 2 digit number (without regrouping)	Throughout everything, solve missing number problems	13 + 4 = 17  14 = 19 - ? 
Add and subtract three 1 digit numbers (make ten first)	Children may need to try different combinations before they find the two numbers that make 10.	 First: 4,7,6 (colours show >10) Second: 4,6,7 Final: total 10,7 so 17 4 + 7 + 6 = 10 + 7 10 = 17

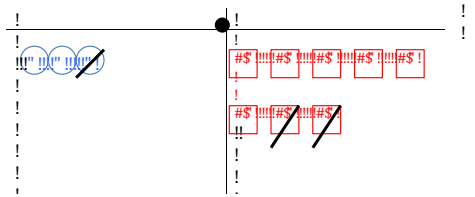
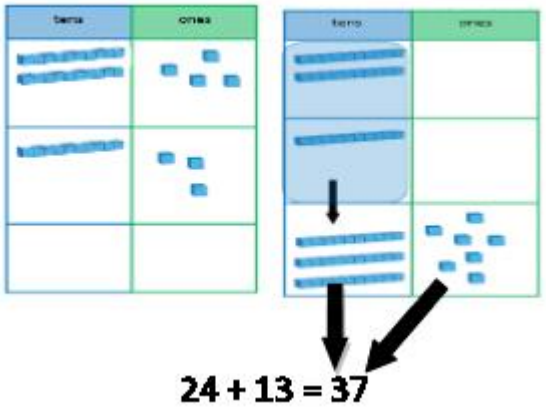
Add a 1 digit number to a 2 digit number; Subtract a 1 digit number from a 2 digit number (with regrouping)	Children may need lots of practice at physically regrouping ten ones for one ten before the two will be seen as interchangeable (e.g. use bundles of straws, dienes)	20 –  4 = ? 6 + 5 = 11 Identify what is needed to make ten first. 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Add and subtract multiples of ten		Using the vocabulary of 1 ten, 2 tens, 3 tens alongside 10, 20, 30 is important because children need to understand that is a ten that is being added/subtracted, not a one. 30 + 20 = 50  6 tens – 2 tens = ____ tens 
Add and subtract near multiples of ten by adding ten and then adjusting		e.g. subtract 9 by subtracting 10 and then adding 1  – 9 = 26

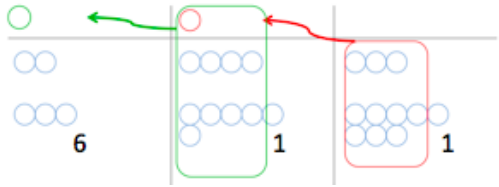
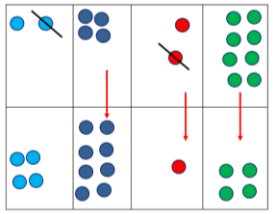
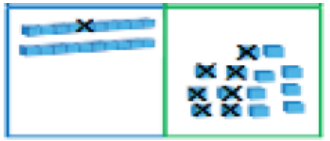
Adding multiples of ten	Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a **ten** that is being added and not a one. It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens'.	

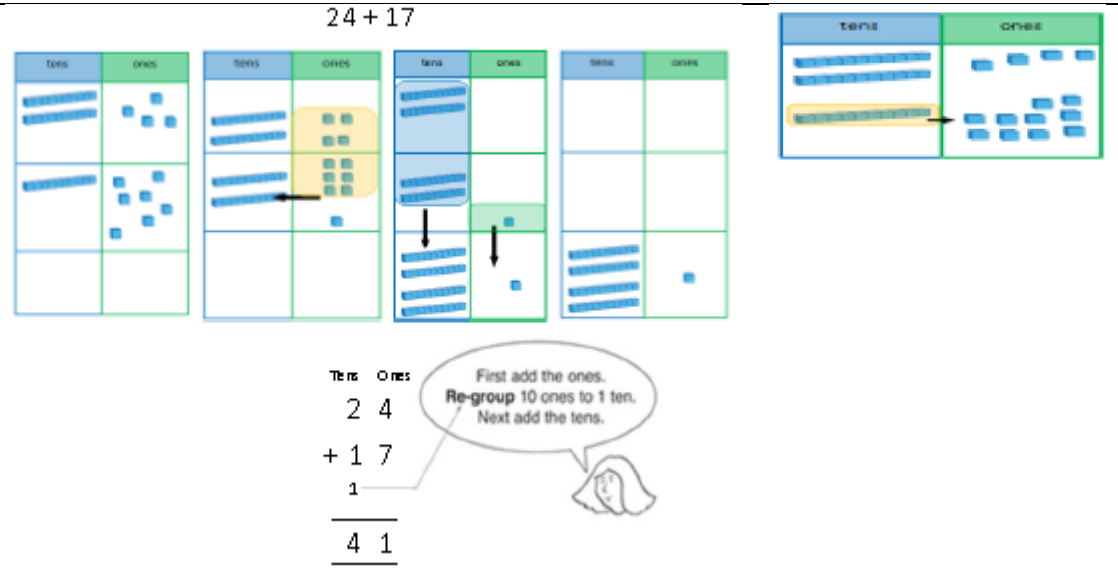
Progression to Formal Methods (mainly Year 2 to Year 6)

Note: Each year the range of numbers to calculate with is extended. Every time, work through the complete sequence described below to ensure children have a deep understanding of *why* the algorithms work, not simply *how* to do them. This ensures children can apply the strategies in unfamiliar problems and increases their accuracy and reliability. For example, when teaching how to subtract decimals, start at step 1, don't just jump straight for the traditional column method and hope children make the connection with their earlier learning.

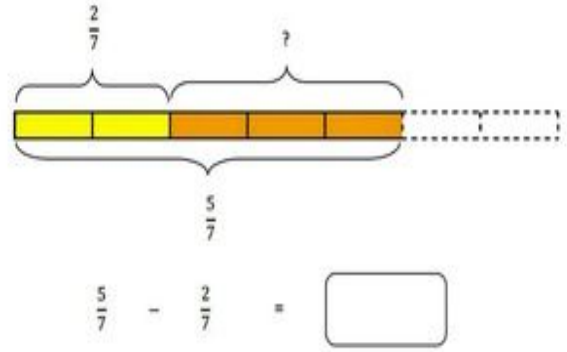
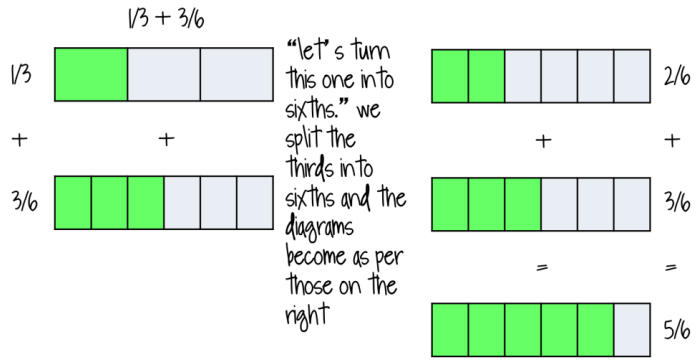
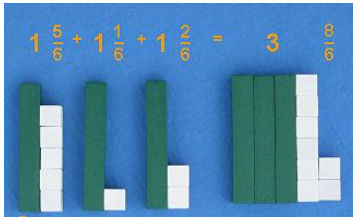
Strategy	Notes	Representations
Step 1: Left to Right Addition/ Subtraction	Place value charts and arrows can help with the partitioning e.g. 	Partition in different ways and then regroup. **47 + 25 = 60 + 12 = 72**  45 – 34 = (40 – 30) + (5 – 4) = 10 + 1 = 11 3.6 + 9.8 (3 + 9) + (0.6 + 0.8) 12 + 1.4 = 13.4
Step 2: Expanded Written Method of Addition/ Subtraction	Start with dienes apparatus and replace with place value counters when children are confident. Then they can move to drawing the apparatus before removing them altogether.	Compare number lines and written method side by side.   200 + 40 + 7 100 + 20 + 5 300 + 60 +12 = 372 247 +125 12 60 300 372

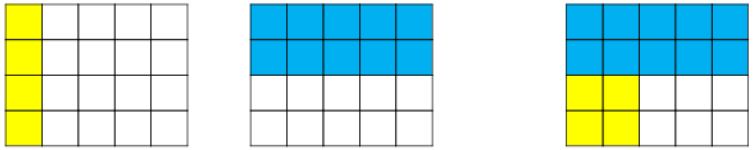
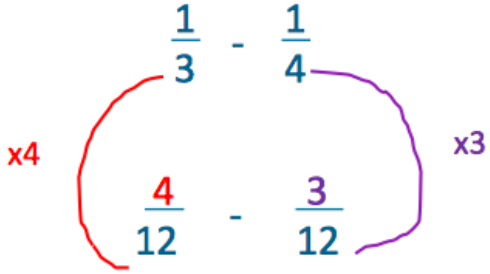
Step 3: Columnar Addition/ Subtraction without regrouping (also called Compact Written Method)	The formal columnar method should be seen as a more streamlined version, not a new method.	3.8 – 1.2 = 2.6  24 + 13 = 37  **24 + 13 = 37**

		The process remains the same no matter how sizeable the numbers become so encourage children to notice the pattern of what is happening and they will be able to extrapolate for much bigger numbers or decimals etc without hesitation. 7 3 3 + 2 1 2 9 4 5	
Step 4: Columnar Addition/ Subtraction with regrouping	If you hear phrases like, "Cross out this and put this," then you need to intervene! We want conceptual understanding not memorising rules without meaning.	Use place value counters, charts and pictures to demonstrate the regrouping first: 1 1 2 4 3 +3 6 8 6 1 1   6232 - 4814 1418	34 – 17 = 17 

		 24 + 17 tens ones Tens Ones 2 4 + 1 7 1 4 1 First add the ones. Re-group 10 ones to 1 ten. Next add the tens.

Adding and Subtracting Fractions (mainly Year 4 to Year 6)

Strategy	Notes	Representations
Add and subtract fractions with the same denominator (Add and subtract fractions with the same denominator within one whole – Y3)	Use bar models and other images to support conceptual understanding	Emphasis that you can only add/subtract with the same noun e.g. 2 cars + 3 cars = 5 cars "2 one-ninths + 3 one-ninths = 5 one-ninths" You can't add 3 pears with 2 apples though! 
Add and subtract fractions with denominators that are multiples of the same number	Link to times tables – use multiplication grids to support finding of common multiples	
Write mathematical statements greater than 1 as a mixed number	Cuisenaire or number rods are great resources to use to explore the relationship between numbers.	 2/5 + 4/5 = 6/5 = 1 1/5

Strategy	Notes	Representations
Add and subtract fractions with different denominators	Use bar models and other images to support conceptual understanding	Start with pictorial approach: $\frac{1}{5} + \frac{2}{4} = \frac{14}{20}$  $\frac{4}{20} + \frac{10}{20} = \frac{14}{20}$ Once the concept is understood, move onto to quicker strategies: 

National Curriculum Progression: Addition and Subtraction

Taken from the NCETM

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Understanding addition and subtraction		*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*
	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot *Understand subtraction as take away and difference (how many more, how many less/fewer)*	*Understand and use take away and difference for subtraction, deciding on the most efficient method for the numbers involved, irrespective of context*			
Addition and subtraction facts	Represent and use number bonds and related subtraction facts within 20	Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 *Recall and use number bonds for multiples of 5 totalling 60 (to support telling time to nearest 5 minutes)*	*Recall and use addition and subtraction facts for 100 (multiples of 5 and 10)* *Derive and use addition and subtraction facts for 100* *Derive and use addition and subtraction facts for multiples of 100 totalling 1000*	*Recall and use addition and subtraction facts for 100* *Recall and use addition and subtraction facts for multiples of 100 totalling 1000* *Derive and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place)*	*Recall and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place)* *Derive and use addition and subtraction facts for 1 (with decimal numbers to two decimal places)*	*Recall and use addition and subtraction facts for 1 (with decimal numbers to two decimal places)*
Mental methods		*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*
	Add and subtract one-digit and two-digit numbers to 20, including zero *(using*	Add and subtract numbers using concrete objects,	Add and subtract numbers mentally, including:	*Add and subtract mentally combinations of two and three digit numbers and*	Add and subtract numbers mentally with increasingly large numbers *and*	Perform mental calculations, including with

	concrete objects and pictorial representations)	pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers	- a three-digit number and ones - a three-digit number and tens - a three-digit number and hundreds	*decimals to one decimal place*	*decimals to two decimal places*	mixed operations and large numbers *and decimals*
Written methods	**Written methods are informal at this stage – see mental methods for expectation of calculations*	**Written methods are informal at this stage – see mental methods for expectation of calculations*	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Add and subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written methods of columnar addition and subtraction where appropriate	Add and subtract whole numbers with more than 4 digits *and decimals with two decimal places*, including using formal written methods (columnar addition and subtraction)	*Add and subtract whole numbers and decimals using formal written methods (columnar addition and subtraction)*
Estimating and checking calculations		Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	Estimate the answer to a calculation and use inverse operations to check answers	Estimate and use inverse operations to check answers to a calculation	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
Order of operations						Use their knowledge of the order of operations to carry out calculations involving the four operations
Solving addition and subtraction problems including those with missing numbers	Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ - 9	Solve problems with addition and subtraction *including those with missing numbers*: - using concrete objects and pictorial representations, including those involving numbers, quantities and measures	Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why *Solve addition and subtraction problems involving missing numbers*	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why *Solve addition and subtraction problems involving missing numbers*	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Solve problems involving addition, subtraction, multiplication and division,

		- applying their increasing knowledge of mental and written methods				*including those with missing numbers*

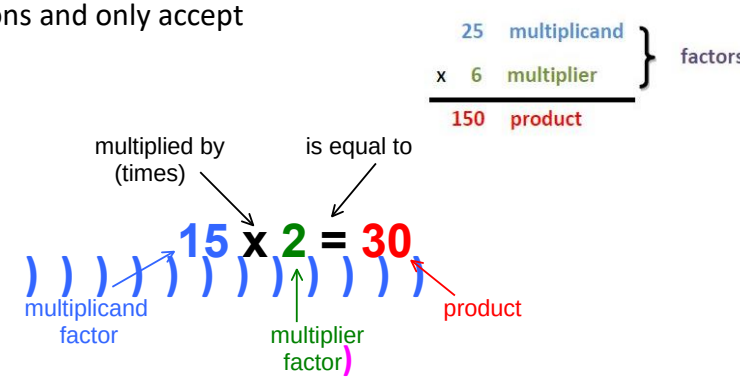
Vocabulary – Multiplication and Division

"Mathematical language is crucial to children's development of thinking. If children don't have the vocabulary to talk about division, or perimeters, or numerical difference, they cannot make progress in understanding these areas of mathematical knowledge."
Mathematical Vocabulary, DfE 2000

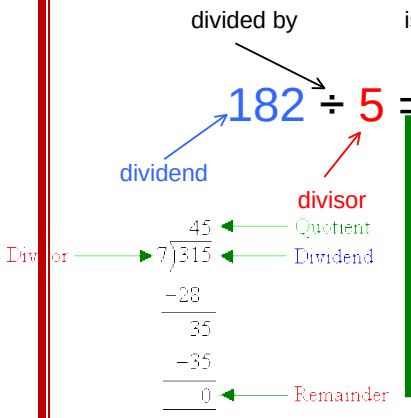
The National Curriculum for Mathematics is very clear that the correct use of mathematical language is central to a meaningful and deep understanding. Having a wide vocabulary of mathematical terminology available is essential for mathematical thinking and reasoning – we think in the same words that we speak. It is not enough for children to simply hear mathematical words; they need to 'feel' them in their own mouths. Therefore when introducing new vocabulary, everyone needs to repeat it out loud. It is also essential that new vocabulary is explained carefully and introduced alongside relevant real life contexts, practical resources or pictures so that children really understand.

Teachers need to have high expectations and only accept what is correct.

✓	×
ones	units
is equal to	equals
zero	oh (the letter O)



Key Vocabulary
Equality, inequality, inverse
Multiplication, multiply, multiplied by
Divide, divided by, divided into
Times, lots of, groups of, once, twice
Ten times, … times as (big, long, wide etc)
Multiple, double, halve, partition, array
Share, share equally, group, equal groups of
Product, factor, common factor/multiple,
Power of, square/cube/prime numbers
Left, left over, remainder

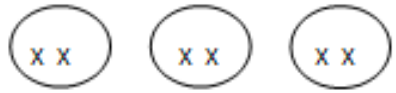


An **equation** is a mathematical statement, in symbols, that says two things are equivalent or the same [number sentence] e.g. 2=10-8, 9-3=6, 5=5

An **inequality** is a mathematical statement that two things are not the same
e.g. 3 < 10 - 9, 12 – 8 > 2 5>2 2<5

Progression in the Early Stages (mainly KS1) MULTIPLICATION AND DIVISION

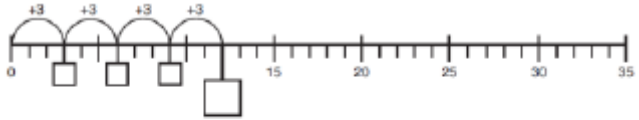
Note: children need a sound understanding of addition (and subtraction) before progressing onto multiplication (and division).

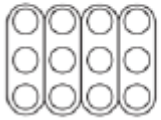
Strategy	Notes	Representations
Linking multiplication to addition through doubling	Use concrete apparatus to show the concept of doubling **not** learning by rote.	Practical resources e.g. Numicon: 
Repeated addition of the same number	Encourage children to read number sentences aloud in different ways e.g. “five times two makes ten”, “ten is equal to five multiplied by two”.	Begin with mostly pictorial representations:  Record this by printing or drawing around Numicon pieces:  Use real life apparatus to count in repeated groups of the same number e.g.

How many wheels are there altogether?

$5+5+5+5+5+5+5+5=$ ☐

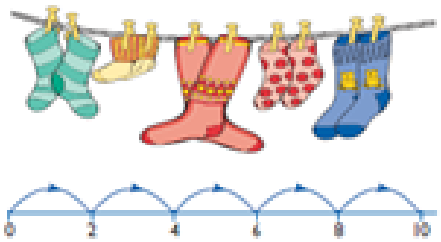
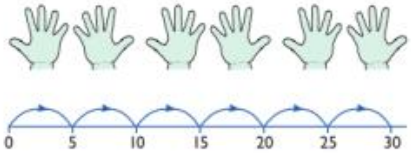
Count in twos, fives and tens aloud and with objects.

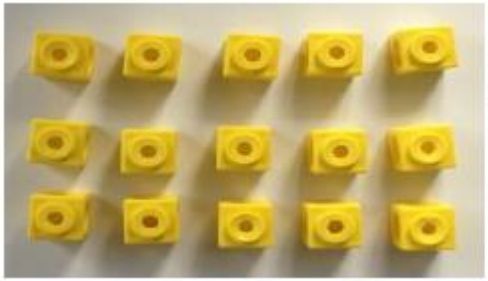
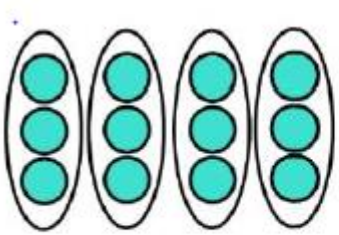
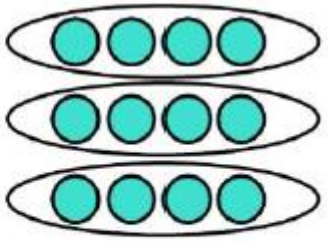
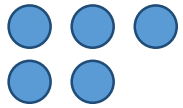
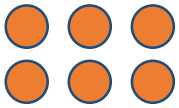


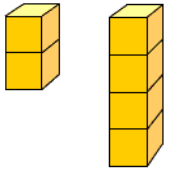
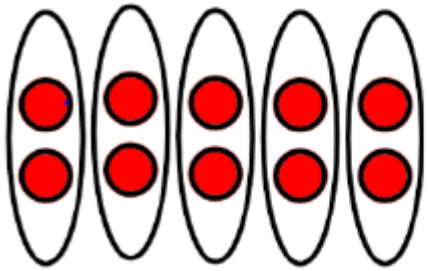
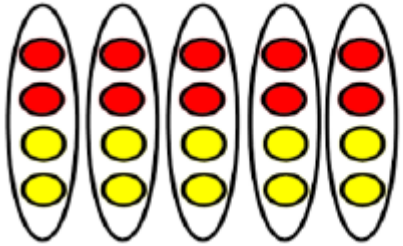
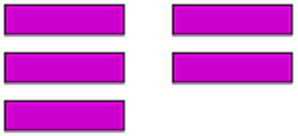


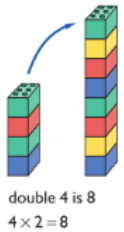
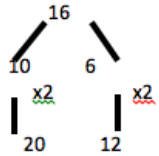
$4 \times 3 =$ ☐

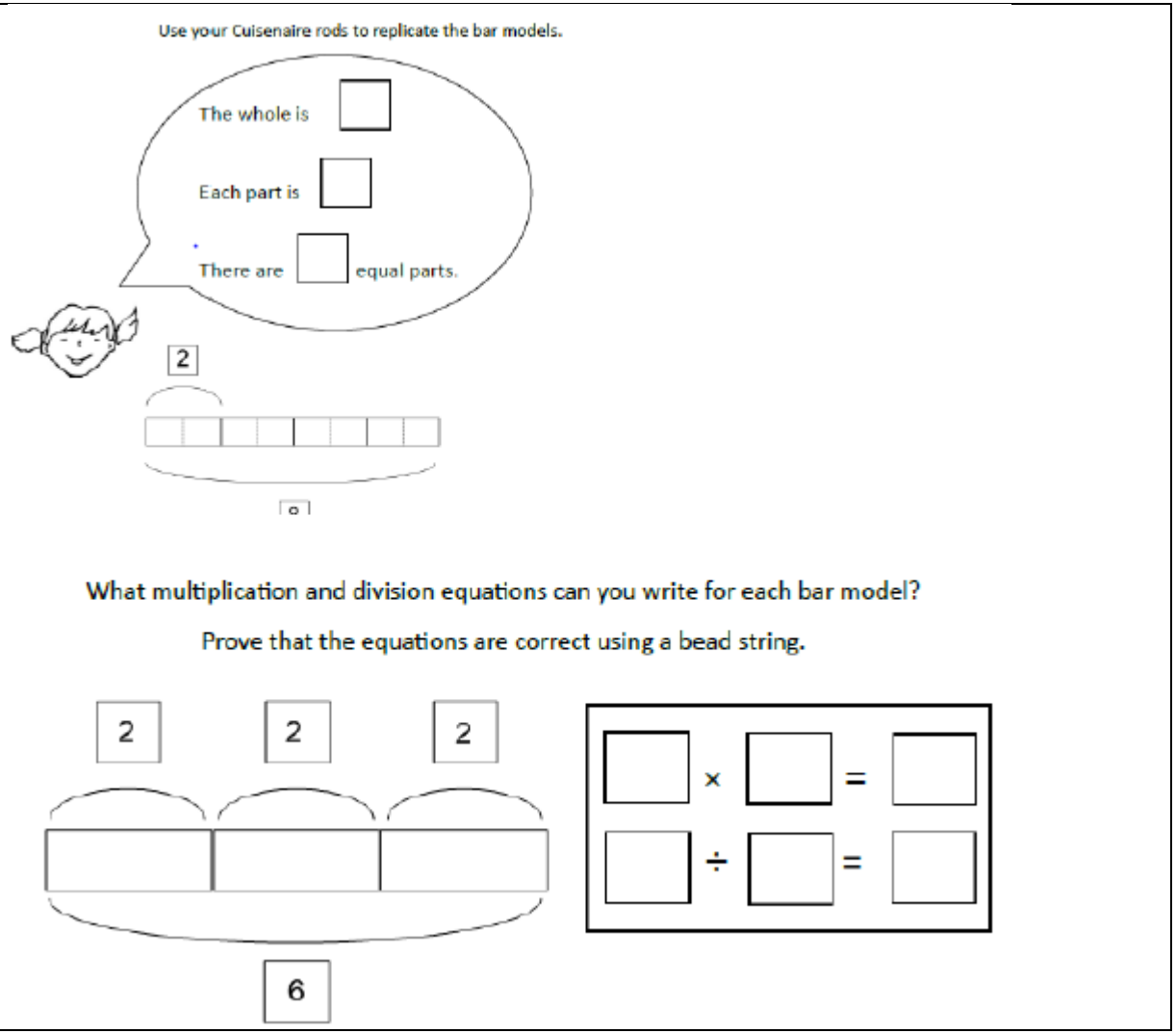
Give multiplication problems set in a real life context and encourage children to visualise the problem e.g. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks? Use number stories to communicate this.

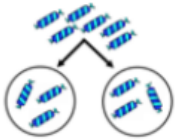
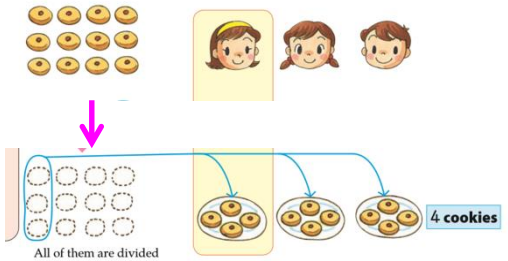
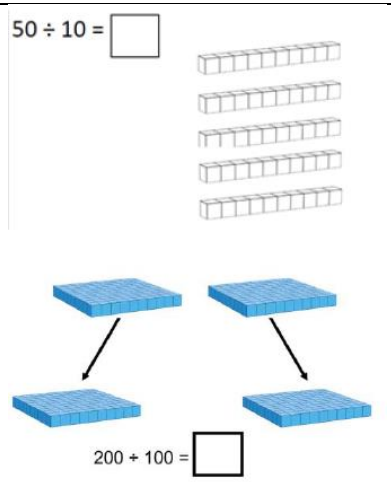
Combing groups of the same size (repeated addition)	Move from addition sentences to multiplication sentences. Ensure the language matches the picture.	Use physical objects and representations such as the number line alongside each other:  $2+2+2+2+2=10$ $2\times5=10$ 2 multiplied by 5 5 pairs 5 hops of 2  $5+5+5+5+5+5=30$ $5\times6=30$ 5 multiplied by 6 6 groups of 5 6 hops of 5
Use arrays to understand multiplication can be done in any order	Multiplication is commutative. Use of concrete manipulatives and images of familiar objects begin to be organised into arrays.	 $4\times2=8$ $2\times4=8$ $2\times4=8$ $4\times2=8$ 2 hops of 4 4 hops of 2

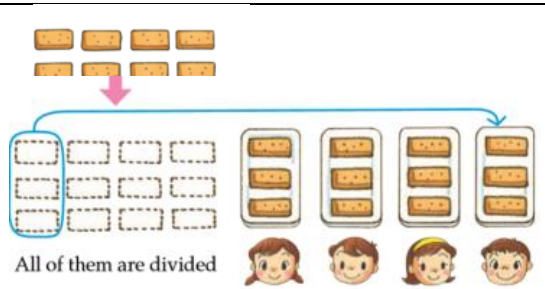
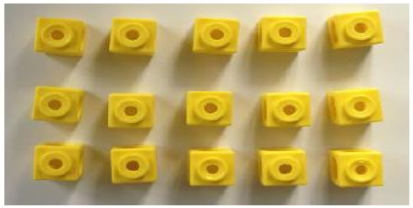
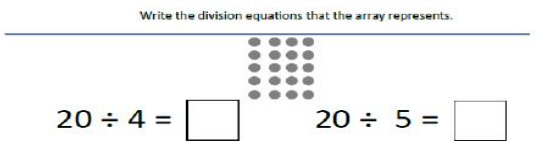
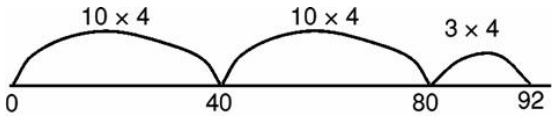
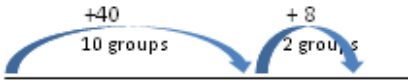
		 3 x 5 = ☐ 5 x 3 = ☐   **12 = 3** × 4 **12 = 4** × 3
Represent odd and even numbers	Seeing this in different ways will help children understand the pattern in numbers.	Use resources e.g. Numicon They can generalise that when counting in 2s all numbers are even → link to 2 times table  *Odd*  *Even*

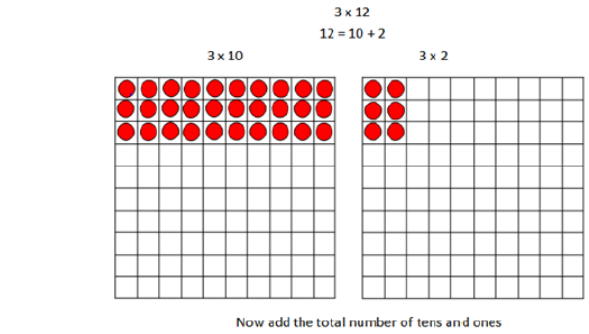
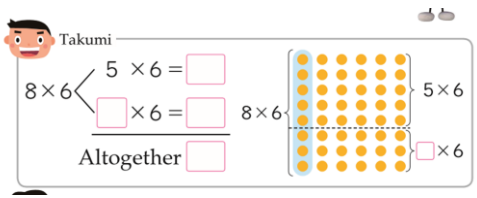
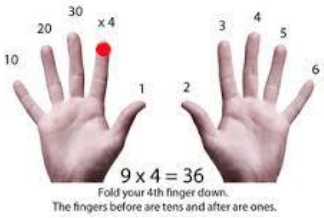
Begin to understand multiplication as scaling in terms of double and half		That tower of cubes is double/half the height of the other tower:   $5 \times 2 = 10$  $5 \times 4 = 20$	Use Cuisenaire and bar models to develop the vocabulary relating to 'times' e.g. Pick up 4, five times  At this stage they double the 2x table facts to derive the 4x table facts.

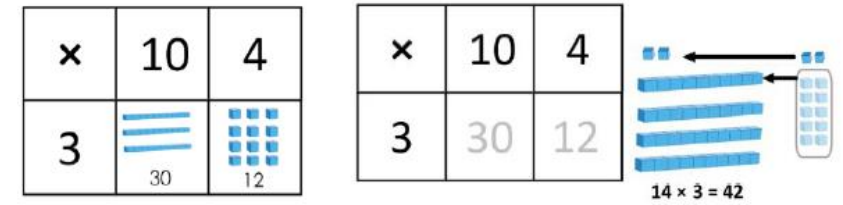
Double numbers up to 10 + 10.			Use known doubles to work out doubling two digit numbers e.g. double 16 = double 10 + double 6 

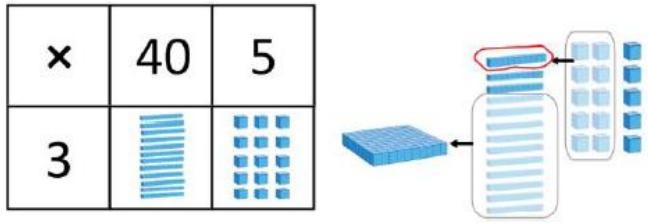
Use of part-part-whole model to establish the inverse relationship between multiplication and division.	This link should be made explicit from early on, using the language of part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should only be used to identify the whole, the size of the parts and the number of the parts.	

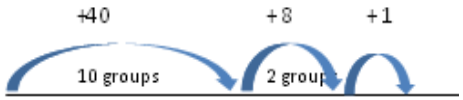
Division as sharing	Introduce intuitively with practical problems.	When 12 cookies are divided evenly among 3 people, 1 child gets 4 cookies. 12 ÷ 3 = 4   Here, if we have 6 fish and they need to be shared between two plates, each plate would hold 3 fish.
Division by partitioning and sharing into equal groups with no regrouping required.	Pupils apply the skills of sharing into equal groups; dividing 2 digit numbers by 3, 4 and 5. They should be encouraged to 'halve' rather than divide by 2.	
Division as grouping	This links to idea of repeated/successive subtraction.	If 12 pastries are divided so each child gets 3, the pastries

		can be shared among 4 people. 12 ÷ 3 = 4  10 fish are divided into groups of 2, so there will be 5 groups. 
Use part-part-whole model to represent division equations and emphasis the relationship between division and multiplication.	Pupils use arrays of concrete manipulatives and images of familiar objects to find division equations. They begin to use dot arrays as a transition to more abstract concept.	  
Efficient use of number lines		Multiplication – using repeated addition of larger amounts e.g. 23 x 4 Division – Children need to be able to partition the dividend in different ways. e.g. 48 **÷ 4 = 12**  

Multiplication of 2 digit numbers with partitioning (no regrouping)	Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for x4) they need to choose the most efficient strategy.	 | × | 10 | 2 | | --- | --- | --- | | 3 | = | ∴ | | × | 10 | 2 | | --- | --- | --- | | 3 | 30 | 6 | **3 x 12 = 36**
Develop fluency with times tables up to 12 x 12	Pupils use multiplication and division as inverses to support the introduction of ratio in Year 6	Develop efficient mental methods: - commutativity and associativity e.g. 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) - distributivity e.g. 39 × 7 = 30 × 7 + 9 × 7 → - derive related facts e.g. 3 × 2 = 6, 6 ÷ 3 = 2 so 30 × 2 = 60, 60 ÷ 3 = 20 - make connections e.g. x12 is double x6 which is double x3 Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?). Use of finger strategy for 9 times table.  

<table>
<tr>
<td>Multiplication of a 2 digit number with partitioning (regrouping)</td>
<td>Using concrete manipulatives and later moving to using images that represent them will support pupils' early understanding, leading towards formal written method.</td>
<td>




</td>
</tr>
<tr>
<td>Introduce remainders</td>
<td>Express results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding e.g. 98 ÷ 4 = 24 r 2 = 24.5 ≈ 25</td>
<td>


49 ÷ 4 = 12 r1
Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49? Left over?

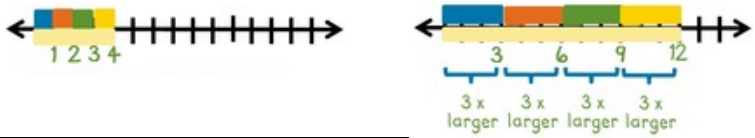
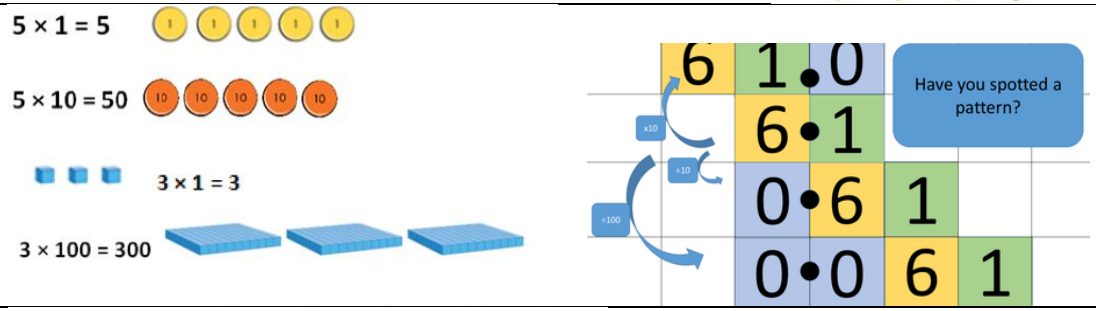
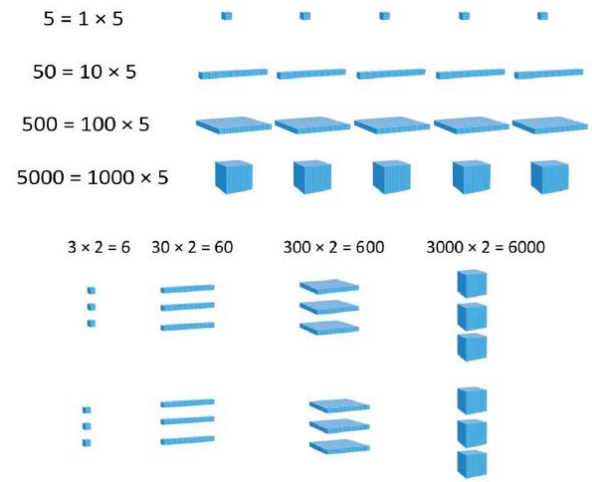
Give opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to be given as a whole number) e.g. pencils are sold in packs of 10. How many packs will I need to buy for 24 children?
</td>
</tr>
<tr>
<td>Recall and use times table facts</td>
<td>Make connections between times tables to help understanding e.g. x5 then x10, x2 then x4</td>
<td>
Regular counting on and back, in steps of 2, 3, 5 and 10. They count in 3 to support their later understanding of a third.

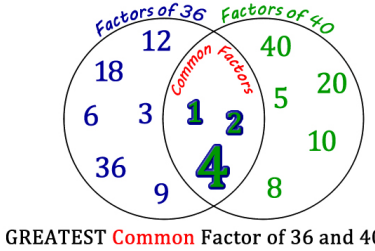
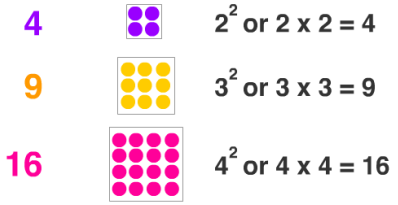
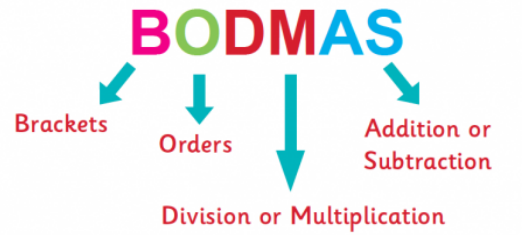
Use a clock face to support understanding of counting in 5s.
Use money to support counting in 2s, 5s, 10s, 20s, 50s.

Use doubling to connect 2, 4 and 8 times tables.


</td>
</tr>
</table>

Calculate mathematical statements for multiplication and division within the times tables	Recognise multiplication and division as inverses through the use of missing number problems.	6 ÷ 2 = □ □ ÷ 2 = 3	□ = 6 ÷ 2 3 = □ ÷ 2	6 ÷ □ = 3 □ ÷ ∇ = 3	3 = 6 ÷ □ 3 = □ ÷ ∇

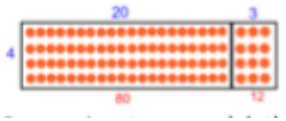
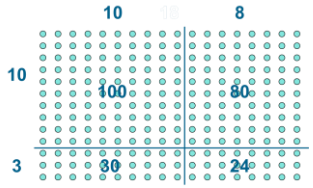
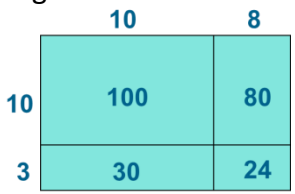
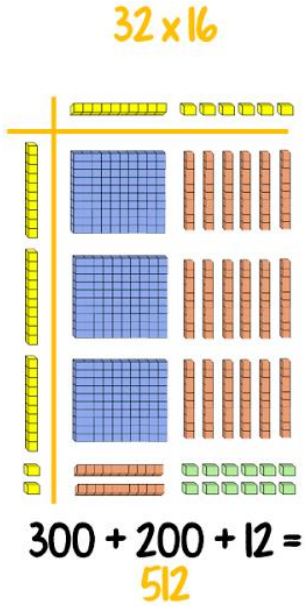
Develop understanding of multiplication as scaling		e.g. 3 times bigger/taller 
Multiply and divide by 10, 100 and 100		
Use known facts for multiplying by multiples of 10, 100 and 100	Pupils' growing understanding of place value allows them to make use of known facts to derive multiplications using powers of 10. Use tables they are familiar with from their year group to show this.	
Further develop understanding of division to find fractions		Use children's intuition to support understanding of fractions as an answer to a sharing problem e.g. 3 apples shared between 4 people = $\frac{3}{4}$ 

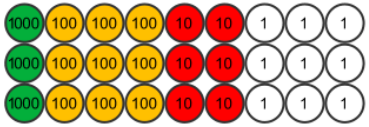
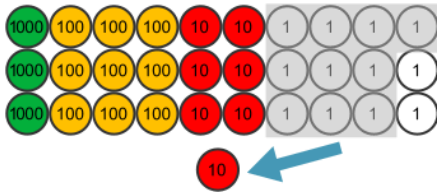
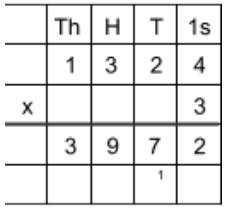
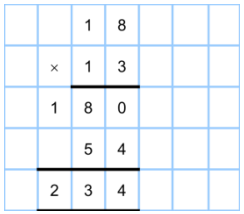
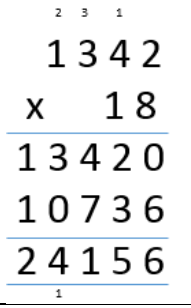
Find common multiples and factors; square numbers and cube numbers; prime numbers	Common factors can be related to finding equivalent fractions	 GREATEST Common Factor of 36 and 40 = 4 
Use the knowledge of the order of operations to calculate with the four operations		Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 

Progression to Formal Methods of Multiplication (mainly Year 3 to Year 6)

Note: Each year the range of numbers to calculate with is extended. Every time, work through the complete sequence described below to ensure children have a deep understanding of *why* the algorithms work, not simply *how* to do them. This ensures children can apply the strategies in unfamiliar problems and increases their accuracy and reliability. For example, when teaching how to multiply decimals, start at step 1, don't just jump straight for the traditional written method and hope children make the connection with their earlier learning.

See Appendix A: Building up to Written Multiplication

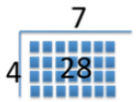
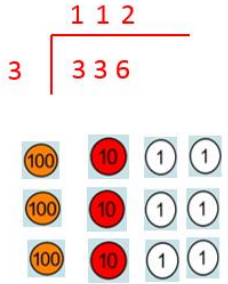
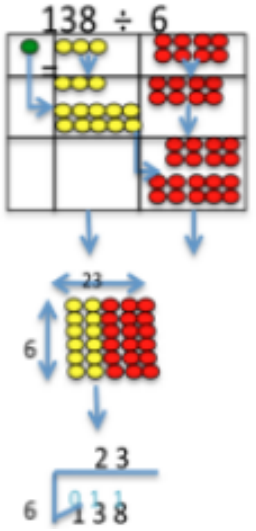
Strategy	Notes	Representations	
Step 1: Partition numbers visually	Lots of practice with physical objects at each level of difficulty is important to ensure conceptual understanding.	Introduce by finding the number of counters in a regular array e.g. 23 x 4 = 92 18 x 13 = 234   Progression to a model that uses the 'area of a rectangle'. Children to draw the rectangles. Often called 'Grid method' e.g. 23 x 4 = 92   	Place value model: 

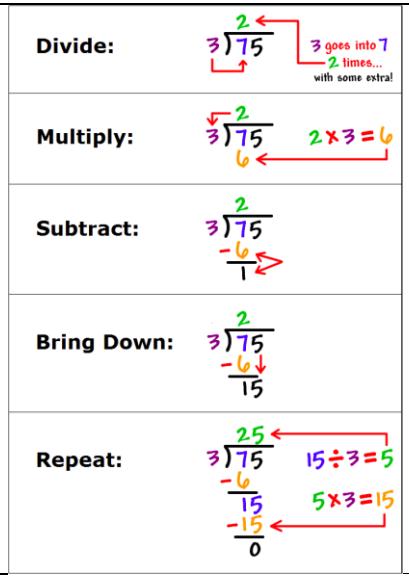
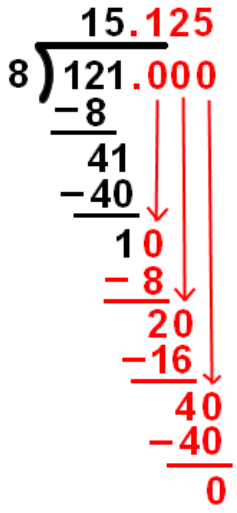
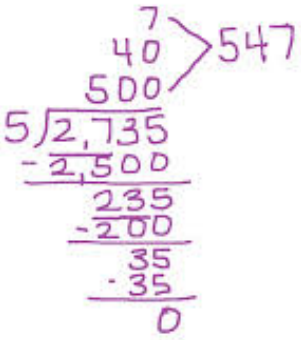
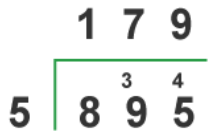
Strategy	Notes	Representations
Step 2: Place Value Counters		Without exchanging e.g. 1323 x 3 – make 3 lots of 1323  With exchanging e.g. 1324 x 3 – make 3 lots of 1324  *Exchange/trade/swap ten 1s for one 10*
Step 3: Expanded Written Method	Compare side by side with pictorial representations – what is the same and different?	234 X 7 28 (4 x 7) 210 (30 x 7) 1400 (200 x 7) 1638 653 x 7 3 x 7 = 21 50 x 7 = 350 + 600 x 7 = 4200 4571
Step 4: Compact Written Method		  

Progression to Formal Methods of Division (mainly Year 4 to Year 6)

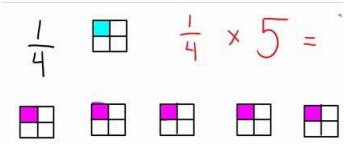
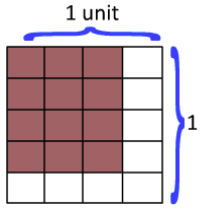
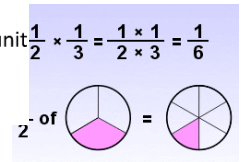
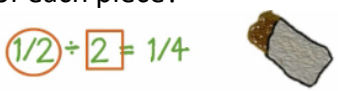
Note: Each year the range of numbers to calculate with is extended. Every time, work through the complete sequence described below to ensure children have a deep understanding of *why* the algorithms work, not simply *how* to do them. This ensures children can apply the strategies in unfamiliar problems and increases their accuracy and reliability. For example, when teaching how to divide decimals, start at step 1, don't just jump straight for the traditional written method and hope children make the connection with their earlier learning.

See Appendix B: Building up to Written Division

Strategy	Notes	Representations
Step 1: Partition numbers visually		Introduce division as an array – physically create the array and draw the bus stop around it  
Step 2: Place Value Counters		Use place value counters to demonstrate what is going on. e.g. exchanging:  

Strategy	Notes	Representations
Step 3: Expanded Written Method – "Long Division"	Compare side by side with pictorial representations – what is the same and different?	  
Step 4: Compact Written Method – "Short Division"	Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)	 

Multiplying and Dividing Fractions (mainly Year 5 and Year 6)

Strategy	Notes	Representations
Multiply proper fractions and mixed numbers by whole numbers	Use bar models and other images to support conceptual understanding before introducing rules	$5 \times \frac{4}{5}$ 5 groups of $\frac{4}{5}$  $\frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} = \frac{20}{5}$  $1\frac{1}{4} \times 3 = \frac{5}{4} \times \frac{3}{1} = \frac{15}{4} = 3\frac{3}{4}$
Associate a fraction with division		Explain how much pizza each person would get if they divided 4 pizzas between 5 people: 4 divided by 5 = $\frac{4}{5}$ = 0.8
Multiply two simple fractions together		$\frac{4}{5} \times \frac{3}{4}$  Across the top, shade in 4 out of 5. Vertically shade in 3 out of 4. Then diagram shows the product: spaces is denominator and shaded is numerator $\frac{12}{20}$ total number of number of spaces 
Divide proper fractions by whole numbers		Ronald and Jamie have ½ candy bar. If ½ a candy bar is split into 2 pieces, what is the size of each piece?  Use 'what do you notice' type questioning to elicit that ¼ ÷ 2 = ¼ x ½

National Curriculum Progression: Multiplication and Division

Taken from the NCETM

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Understanding multiplication and division			*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known or related fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known or related fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known or related fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known or related fact, calculate mentally, use a jotting, written method)*
		Understand multiplication as repeated addition *Understand division as sharing and grouping and that a division calculation can have a remainder* Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot	*Understand that division is the inverse of multiplication and vice versa* *Understand how multiplication and division statements can be represented using arrays* *Understand division as sharing and grouping and use each appropriately*	Recognise and use factor pairs and commutativity in mental calculations	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers	
Multiplication and division facts		Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	Recall multiplication and division facts for multiplication tables up to 12 × 12	Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers Establish whether a number up to 100 is prime and recall prime numbers up to 19 Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	Identify common factors, common multiples and prime numbers
	Recall and use doubles of all numbers to 10 and corresponding halves	*Derive and use doubles of simple two-digit numbers*	*Derive and use doubles of all numbers to 100 and corresponding halves*	*Use partitioning to double or halve any number, including*	*Use partitioning to double or halve any number, including*	*Use partitioning to double or halve any number*

		(numbers in which the ones total less than 10) *Derive and use halves of simple two-digit even numbers (numbers in which the tens are even)*	*Derive and use doubles of all multiples of 50 to 500*	*decimals to one decimal place*	*decimals to two decimal places*	
Mental methods		Calculate mathematical statements for multiplication *(using repeated addition)* and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods	Use place value, known and derived facts to multiply and divide mentally, including: - multiplying by 0 and 1 - dividing by 1 - multiplying together three numbers	Multiply and divide numbers mentally drawing upon known facts Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes	Perform mental calculations, including with mixed operations and large numbers
Written methods	**Written methods are informal at this stage – see mental methods for expectation of calculations*	**Written methods are informal at this stage – see mental methods for expectation of calculations*	Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication Multiply one-digit numbers with up to two decimal places by whole numbers
			Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods	*Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context*	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context Use written division methods in cases where the answer has up to two decimal places
Estimating and checking calculations			*Use estimation to check answers to calculations and determine, in the context of*	*Use estimation and inverse to check answers to calculations and determine, in the context*	*Use estimation and inverse to check answers to calculations and determine,*	Use estimation *and inverse* to check answers to calculations and determine,

			a problem, an appropriate degree of accuracy	*of a problem, an appropriate degree of accuracy*	*in the context of a problem, an appropriate degree of accuracy*	in the context of a problem, an appropriate degree of accuracy
Order of operations						Use their knowledge of the order of operations to carry out calculations involving the four operations
Solving multiplication and division problems including those with missing numbers	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	Solve problems involving multiplication and division *(including those with remainders),* using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	Solve problems, including missing number problems, involving multiplication and division *(and interpreting remainders)*, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, *division (including interpreting remainders),* integer scaling problems and harder correspondence problems such as n objects are connected to m objects	Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	Solve problems involving addition, subtraction, multiplication and division